

M250

Practice sections 1.1-1.5

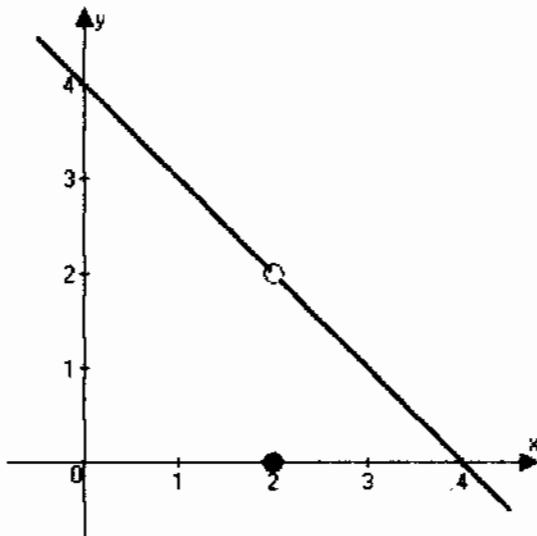
Name: Key Date: _____

1. Let

$$f(x) = \begin{cases} 4-x, & x \neq 2 \\ 0, & x = 2 \end{cases}$$

Determine the following limit. (Hint: Use the graph of the function.)

$$\lim_{x \rightarrow 2} f(x) = 2$$

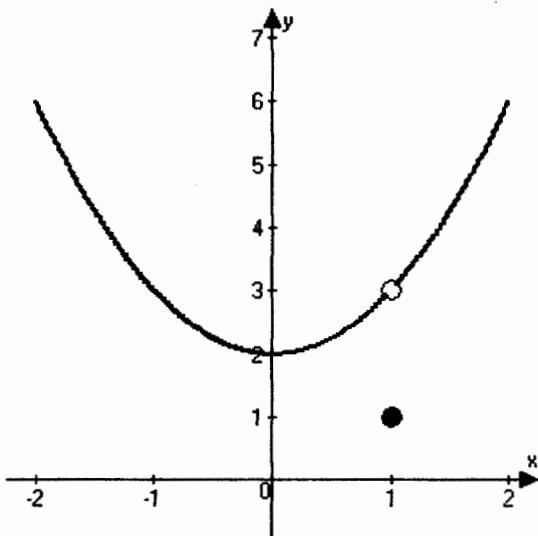


2. Let

$$f(x) = \begin{cases} x^2 + 2, & x \neq 1 \\ 1, & x = 1 \end{cases}$$

Determine the following limit. (Hint: Use the graph of the function.)

$$\lim_{x \rightarrow 1} f(x) = 3$$



3. Let $f(x) = 4x + 3$ and $g(x) = x^3$. Find the limits:

(a) $\lim_{x \rightarrow 3} f(x) = 15$ (b) $\lim_{x \rightarrow 5} g(x) = 125$ (c) $\lim_{x \rightarrow 5} g(f(x)) = 23^3 = 12,167$

4. Let $f(x) = x^2 - 3$ and $g(x) = 2x$. Find the limits:

(a) $\lim_{x \rightarrow -1} f(x) = -2$ (b) $\lim_{x \rightarrow -3} g(x) = -6$ (c) $\lim_{x \rightarrow -4} g(f(x)) = 26$

5. Let $f(x) = 3 + x^2$ and $g(x) = \sqrt{x+2}$. Find the limits:

(a) $\lim_{x \rightarrow 3} f(x) = 12$ (b) $\lim_{x \rightarrow 3} g(x) = \sqrt{5}$ (c) $\lim_{x \rightarrow 3} g(f(x)) = \sqrt{14}$

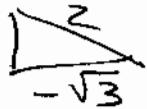
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6. Let $f(x) = 4x^2 - 5x - 4$ and $g(x) = \sqrt[3]{x-5}$. Find the limits:

(a) $\lim_{x \rightarrow 5} f(x) = 71$ (b) $\lim_{x \rightarrow 1} g(x) = -\sqrt[3]{4}$ (c) $\lim_{x \rightarrow 2} g(f(x)) = -\sqrt[3]{3}$

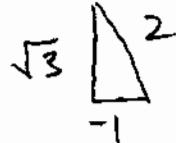
7. Find the limit:

$$\lim_{x \rightarrow \frac{5\pi}{6}} \sin x = \frac{1}{2}$$



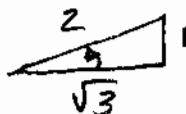
8. Find the limit:

$$\lim_{x \rightarrow 2} \cos\left(\frac{\pi x}{3}\right) = -\frac{1}{2}$$



9. Find the limit:

$$\lim_{x \rightarrow \pi} \tan\left(\frac{x}{6}\right) = \frac{1}{\sqrt{3}}$$



10. Suppose that $\lim_{x \rightarrow c} f(x) = 7$ and $\lim_{x \rightarrow c} g(x) = 6$. Find the following limit:

$$\lim_{x \rightarrow c} [f(x)^{g(x)}] = 7^6$$

11. Suppose that $\lim_{x \rightarrow c} f(x) = 15$ and $\lim_{x \rightarrow c} g(x) = -7$. Find the following limit:

$$\lim_{x \rightarrow c} [f(x) + g(x)] = 15 - 7 = 8$$

12. Suppose that $\lim_{x \rightarrow c} f(x) = -12$ and $\lim_{x \rightarrow c} g(x) = -8$. Find the following limit:

$$\lim_{x \rightarrow c} [f(x) - g(x)] = -12 - -8 = -4$$

13. Suppose that $\lim_{x \rightarrow c} f(x) = -8$ and $\lim_{x \rightarrow c} g(x) = 4$. Find the following limit:

$$\lim_{x \rightarrow c} [-9g(x)] = -9 \cdot 4 = -36$$

14. Suppose that $\lim_{x \rightarrow c} f(x) = 6$ and $\lim_{x \rightarrow c} g(x) = -2$. Find the following limit:

$$\lim_{x \rightarrow c} [f(x)g(x)] = 6(-2) = -12$$

15. Suppose that $\lim_{x \rightarrow c} f(x) = 11$ and $\lim_{x \rightarrow c} g(x) = -9$. Find the following limit:

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = -\frac{11}{9}$$

16. Find the following limit (if it exists). Write a simpler function that agrees with the given function at all but one point.

$$\lim_{x \rightarrow -6} \frac{x^3 + 216}{x + 6} = \lim_{x \rightarrow -6} \frac{(x+6)(x^2 - 6x + 36)}{(x+6)} = \lim_{x \rightarrow -6} (x^2 - 6x + 36) = 108$$

17. Find the following limit (if it exists). Write a simpler function that agrees with the given function at all but one point.

$$\lim_{x \rightarrow 1} \frac{-3x^2 + 14x - 11}{x - 1} = \lim_{x \rightarrow 1} \frac{(x-1)(-3x+11)}{(x-1)} = \lim_{x \rightarrow 1} -3x + 11 = 8$$

18. Find the limit (if it exists):

$$\lim_{x \rightarrow 4} \frac{x+4}{x^2 - 16} = \lim_{x \rightarrow 4} \frac{(x+4)}{(x+4)(x-4)} = \lim_{x \rightarrow 4} \frac{1}{(x-4)} = -\frac{1}{8}$$

19. Find the limit (if it exists):

$$\lim_{\Delta x \rightarrow 0} \frac{(x+\Delta x)^2 + (x+\Delta x) + 1 - (x^2 + x + 1)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 + x + \Delta x - x^2 - x - 1}{\Delta x} = \lim_{\Delta x \rightarrow 0} 2x + \Delta x + 1 = 2x + 1$$

20. Determine the limit (if it exists):

$$\lim_{x \rightarrow 0} \frac{\sin x (1 - \cos x)}{3x^6} (1 + \cos x) = \lim_{x \rightarrow 0} \frac{\sin^3 x}{x^3} \cdot \frac{1}{x^3} \Rightarrow \text{No Limit}$$

21. Determine the limit (if it exists):

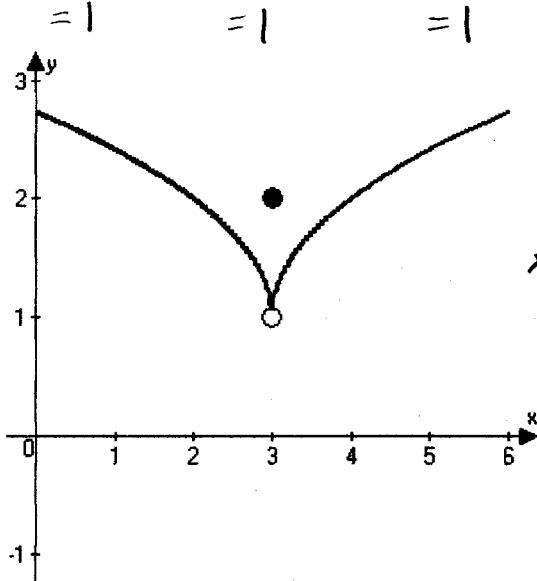
$$\lim_{x \rightarrow 0} \frac{-2(1 - \cos x)}{x^2} = \lim_{x \rightarrow 0} \frac{-2(1 - \cos x)(1 + \cos x)}{x^2 (1 + \cos x)} = -2 \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2 \cdot 2} = -1$$

22. Determine the limit (if it exists):

$$\lim_{x \rightarrow 0} \frac{\sin^6 x}{x^6} = \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right)^6 = 1$$

23. Use the graph as shown to determine the following limits, and discuss the continuity of the function at $x = 3$.

$$(i) \lim_{x \rightarrow 3^+} f(x) \quad (ii) \lim_{x \rightarrow 3^-} f(x) \quad (iii) \lim_{x \rightarrow 3} f(x)$$

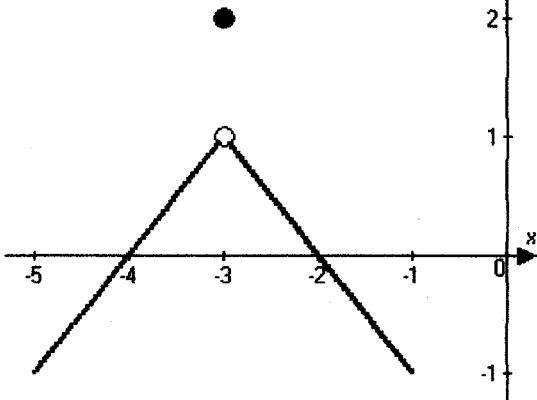


Function is discontinuous
at $x = 3$ because
 $\lim_{x \rightarrow 3} f(x) = 1 \neq 2 = f(3)$

24. Use the graph as shown to determine the following limits, and discuss the continuity of the function at $x = -3$.

$$(i) \lim_{x \rightarrow -3^+} f(x) \quad (ii) \lim_{x \rightarrow -3^-} f(x) \quad (iii) \lim_{x \rightarrow -3} f(x)$$

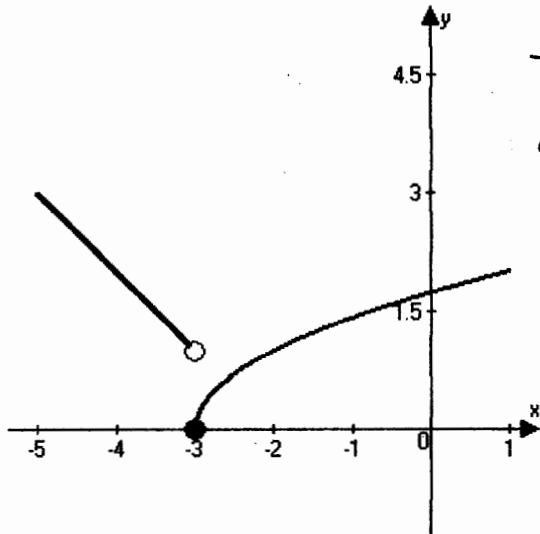
$$= 1 \quad = 1 \quad = 1$$



Function is discontinuous
at $x = 3$ because
 $\lim_{x \rightarrow 3} f(x) = 1 \neq 2 = f(3)$

25. Use the graph to determine the following limits, and discuss the continuity of the function at $x = -3$.

$$\begin{array}{lll} \text{(i)} \lim_{x \rightarrow -3^+} f(x) & \text{(ii)} \lim_{x \rightarrow -3^-} f(x) & \text{(iii)} \lim_{x \rightarrow -3} f(x) \\ = 0 & = 1 & \text{D.N.E.} \end{array}$$



The function is discontinuous at $x = -3$ because $\lim_{x \rightarrow -3} f(x)$ does not exist.

26. Find the x -values (if any) at which the function $f(x) = -14x^2 - 14x - 9$ is not continuous. Which of the discontinuities are removable? continuous for all x .

27. Find the x -values (if any) at which the function $f(x) = \frac{x}{x^2 - 49}$ is not continuous. $\{ -7, 7 \}$

Which of the discontinuities are removable? NO NONE

28. Find the x -values (if any) at which the function $f(x) = \frac{x-3}{x^2 - 9x + 18}$ is not continuous.

Which of the discontinuities are removable?

29. Find constants a and b such that the function

$$f(x) = \begin{cases} 8, & x \leq -7 \\ ax + b, & -7 < x < 9 \\ -8, & x \geq 9 \end{cases}$$

is continuous on the entire real line.

$$f(9) = -8 = \lim_{x \rightarrow 9} f(x) \Rightarrow a(9) + b = -8$$

$$\Rightarrow 16a = -16 \Rightarrow a = -1$$

$$\Rightarrow b = 1$$

30. Find the constant a such that the function

$$f(x) = \begin{cases} -7 \cdot \frac{\sin x}{x}, & x < 0 \\ a + 9x, & x \geq 0 \end{cases}$$

$$f(0) = a = \lim_{x \rightarrow 0^-} f(x) = -7$$

$\Rightarrow a = -7$

is continuous on the entire real line.

31. Find the vertical asymptotes (if any) of the function $f(x) = \frac{x^2 - 100}{x^2 + 4x - 60}$.

$$\frac{(x-10)(x+10)}{(x-6)(x+10)}$$

$\Rightarrow x=6$ is a VA

32. Find the vertical asymptotes (if any) of the function $f(x) = \frac{x^2 + 4x + 3}{x^3 - 7x^2 + 7x + 15}$.

$$\frac{(x+3)(x+1)}{(x^2 - 8x + 15)(x+1)}$$

33. Find the vertical asymptotes (if any) of the function $f(x) = \tan(-15x)$.

34. Find the limit:

$$\lim_{x \rightarrow 7^+} \frac{x+10}{x-7} = \infty$$

$\frac{\sin(-15x)}{\cos(-15x)}$ VA for
 $\cos(-15x) = 0$
 $\Rightarrow -15x = (2n+1)\frac{\pi}{2}$
 $\Rightarrow x = \frac{(2n+1)\pi}{30} = \frac{(2m+1)\pi}{30}$

$\Rightarrow x = 3$ and $x = 5$ are vertical asymptotes

35. Find the limit:

$$\lim_{x \rightarrow 12} \frac{x^2 - 12x}{(x^2 + 144)(x-12)} = \lim_{x \rightarrow 12} \frac{x}{x^2 + 144} = \frac{1}{24}$$

36. Find the limit:

$$\lim_{x \rightarrow 0^-} \left(x^9 + \frac{1}{x} \right) = -\infty$$